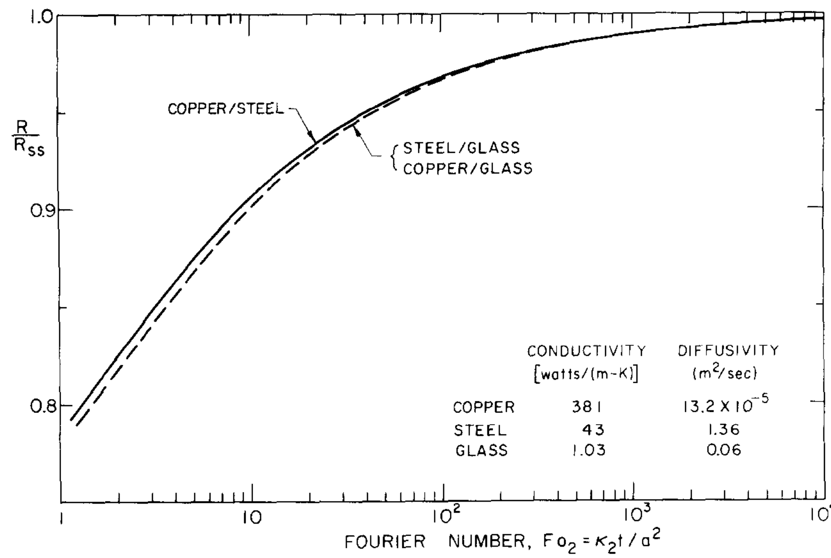

FIG. 1. Unsteady thermal resistances of different pairs of solids as a function of the Fourier number.

all the four cases lie close to a single curve. This curve, however, is not consistent with the individual plots they obtained, but it agrees approximately with equation (16) of the present study for Fourier numbers greater than unity.

The thermal resistance, in general, depends on the conductivity ratio, k_1/k_2, the diffusivity ratio, κ_1/κ_2, and the Fourier number $\kappa_2 t/a^2$, where we take $\kappa_2 < \kappa_1$. Schneider *et al.* [2] showed that for the cases they considered, the influence of the conductivity ratio appeared only in the steady-state part of the thermal resistance. They concluded that when the resistance is normalized with the steady-state value, the result depends only on the diffusivity ratio and the Fourier number. In the present study, however, equation (16) shows that the conclusion of Schneider *et al.* [2] is not necessarily valid. The results of Schneider *et al.* [2] showing no dependence on k_2/k_1 is not surprising, because they considered materials which, like most solids, have $k_2/k_1 \simeq \kappa_2/\kappa_1$. The present study, however, is completely general and it exposes the dependence on both k_2/k_1 and κ_2/κ_1.

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ON THE TRANSITION TO TRANSVERSE ROLLS IN INCLINED INFINITE FLUID LAYERS — STEADY SOLUTIONS

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NOMENCLATURE

Gr, Grashof number;
H, spacing between plates;
Pr, Prandtl number;
x, y, z, spatial coordinates;
T, temperature;
T_0, reference temperature.

Greek

α, spatial wavenumber;

ΔT, temperature difference of plates;
ϕ, angle of inclination.

Subscript

c, critical condition.

1. INTRODUCTION

1.1. *The problem*

THIS paper describes results of a study on the stability of the conduction regime of fluids in inclined, infinitely long slots, to disturbances with their axes in the horizontal. The fluid is contained between two parallel, flat, isothermal, rigid plates of essentially infinite area, separated by distance H, and tilted at an angle ϕ from the horizontal. The temperature difference of the plates is ΔT; one plate having the temperature $T_0 - \Delta T/2$, and the other plate having the temperature $T_0 + \Delta T/2$. It is well known that provided the Grashof number, Gr, is larger than zero, an imbalance between pressure and buoyancy forces causes fluid motion to always exist. At low Gr, this motion is described as a base flow, the velocity being cubic in z, the direction perpendicular to the plates. The corresponding temperature is linear in z and heat transfer is by conduction (hence the regime name). At sufficiently large Gr, the conduction regime becomes unstable and suffers a transition to the multicellular regime. This transition is the subject of the present paper, with exclusive consideration given to transitions resulting in rolls with their axes in the horizontal (termed transverse rolls).

The literature concerning the stability of the conduction regime in the inclined case is quite sparse. The first works for such layers were by Birikh *et al.* [1] and Gershuni and Zhukhovitskii [2]. The results of the second paper are discussed more fully in the book by Gershuni and Zhukhovitskii [3]. Based on calculations for $0° \leq \phi \leq 90°$, in 10° increments, and the Pr numbers 0.2, 1.0 and 5.0, they concluded that: for $0° \leq \phi \leq 40°$, the critical condition is defined by the critical Rayleigh number, Ra_c, it being essentially constant for the three Pr numbers; for $40° \leq \phi \leq 90°$, the critical condition is defined by the critical Grashof number, Gr_c, it being essentially constant for the three Pr numbers; the change of behaviour at $\phi \simeq 40°$ occurs smoothly; and for all angles the critical wavenumber, α_c, is practically independent of Pr and only slightly sensitive to changes in ϕ.

Hart [4], as part of an extensive study of flow in inclined layers, calculated critical Rayleigh numbers for $Pr = 0.70$ and 6.7. The most recent theoretical work on the problem is due to Korpela [5]. He concluded that, for $0.24 \leq Pr \leq 12.7$ and ϕ close to 90°, transverse rolls occur. As ϕ decreases, the transverse rolls are replaced by rolls whose axes are parallel-up the plates (longitudinal rolls). The angle at which this change occurred was Pr dependent and graphical results were presented to describe this dependence. For $Pr < 0.24$, it was shown that transverse rolls always occur, regardless of the angle. The only experimental values of the critical conditions for transverse rolls in inclined fluid layers known to the author are by Hollands and Konicek [6]. These results are based on heat-transfer measurements, with the critical condition, Ra_c, found by detecting a point where the Nusselt number, Nu, becomes larger than 1. Accurate determination of Ra_c depended on fitting an assumed heat-transfer relationship, of the form $Nu = Nu(Ra)$, to the heat-transfer data points in order to locate the intersection point with $Nu = 1$. The experiments were performed using air as the working fluid; angles between 0° and 90° were studied. Different values of Ra_c were obtained depending on the equation used to fit the data.

The present paper describes the results of a stability analysis over the range $0 \leq Pr \leq 10$ and the angles $10° \leq \phi \leq 80°$. Results were obtained using the power series method described by Ruth [7]. These results comprise the calculation of the stability conditions for 809 different combinations of Pr and ϕ.

2.1. *The results*

The results are summarized in Figs. 1–4. The Gr_c numbers of [3] are plotted for comparison; agreement with these workers is good. A comparison of the present results with the experiments of [6] is presented in Table 1. Agreement is good for $\phi = 75°$, fair for $\phi = 80°$, and poor for 85° and 90°. The best correlation is obtained by using their equation (6), a linear fit of Nu with Ra.

Transverse rolls will occur only if their Ra_c is less than $1707.762/\cos\phi$. Therefore, although the critical conditions for transverse rolls may be calculated for any combination of ϕ and Pr, only certain of these transitions will actually occur. In Figs. 1–4, critical conditions which are physically realizable are plotted along solid curves, while those which will not physically occur, since they are preceded by the longitudinal roll condition, are plotted along broken curves. The Pr below which transverse rolls always occur (regardless of angle) is 0.26, which is in essential agreement with the result of [5].

As previously noted by [3], the stability criteria behaviour with angle may be separated into two angle-range regions, depending on whether the criteria are best characterized by a Gr_c or a Ra_c. As a result of the present computations, both their conclusions as to the angle-range over which either Gr_c or Ra_c is preferred, and their description of the variations of Gr_c and Ra_c with Pr within these ranges, must be modified. For $\phi \leq 20°$, not $\phi \leq 40°$ as previously concluded, the stability criteria are best characterized by Ra_c. Furthermore, although in the Pr range studied in [3] $(0.2 \leq Pr \leq 5)$ Ra_c is essentially constant, from Fig. 1 it is clear that, as $Pr \to 0$, Ra_c varies substantially. In the other limit, as $Pr \to 10$, Ra_c does approach a limit. For $\phi \geq 30°$, the stability condition is best characterized by a Gr_c*. Once again the conclusion in [3] that, at any one angle, Gr_c is essentially constant with Pr, does not hold. As $Pr \to 0$, there is substantial variation in Gr_c, as can be verified by inspecting Fig. 3. For $\phi \geq 30°$, as $Pr \to 10$, the Gr_c appears to be approaching a limit. Furthermore, the transition between the two characterized regions is not smooth as stated in [3]. The results for $\phi = 25°$, illustrated in Fig. 1, clearly show that the transition between the two regions depends strongly on Pr and ϕ. Whereas the results for $\phi = 20°$ and 30° show smooth variations in Ra_c with Pr, at $\phi = 25°$ and $Pr \simeq 2.15$, a virtual discontinuity in Ra_c occurs. For $Pr < 2.15$, the $\phi = 20°$ curve appears to be analogous to the $\phi = 20°$ curve, while for $Pr > 2.15$ the $\phi = 25°$ curve appears to be analogous to the $\phi = 30°$ curve. It is further concluded in [3] that α_c is not very dependent on either Pr or ϕ. In fact, as illustrated in Fig. 2, some very interesting dependencies exist. For $\phi = 10°$, α_c rises from its $Pr = 0$ limit of 2.688, to a value at $Pr = 10$ very near the horizontal result of 3.116. For $\phi = 20°$, the same general behaviour is exhibited; however, in the region of $Pr = 1.0$, a flattening of the curve is apparent. For $\phi = 30°$, this flattening has developed into a bimodal curve, reminiscent of the $\phi = 90°$ results described in [7]. As illustrated in Fig. 4, this bimodal behaviour is characteristic of all angles larger than 30°. For $\phi = 25°$ and $Pr \simeq 2.15$, a discontinuity in α_c is exhibited, analogous to the discontinuity in Ra_c at this same point. For $\phi > 30°$, the behaviour of α_c with Pr is plotted in Fig. 4. All the curves have a similar shape and there is a continuous shift in the location of extrema as ϕ increases.

For $\phi \geq 30°$, the behaviour of Gr_c and α_c with Pr is similar to that already described in [7] for $\phi = 90°$. Fig. 3 shows the Pr location of extrema shifting to higher values as ϕ decreases from 80 to 30°. The behaviour of the α_c curves are similar, with their extrema also shifting to higher values of Pr as ϕ decreases. As $Pr \to 0$, a limiting case exists for equation (1). The critical conditions for these limiting equations is known to yield the results (see for example Hart [8])

$$\alpha_c = 2.688, \quad Gr_c \sin\phi = 7930.055.$$

These conditions are essentially satisfied for all angles provided $Pr < 10^{-3}$.

*The results are plotted in the form $Gr_c \sin\phi$ since this allows them to be collapsed onto a single graph.

Table 1. Comparison of present Ra_c with experiments of Hollands and Konicek [6]

		Hollands and Konicek [6]		
ϕ	Present	equation (5)	equation (6)	equation (7)
75	5396.7	6215 ± 487	5561 ± 653	5990 ± 506
80	5464.1	6547 ± 761	5974 ± 562	6330 ± 833
85	5566.0	6942 ± 178	6454 ± 819	6750 ± 383
90	5706.7	7861 ± 363	7370 ± 365	7664 ± 446

3. CONCLUSIONS

The conclusions of the present study are as follows:

1. By means of the power series method, essentially exact solutions of Gr_c and α_c, as functions of Pr, have been found for angles of inclination ranging over $10° \leq \phi \leq 80°$.
2. The present study confirms the critical conditions calculated by Gerushuni and Zhukhovitskii [2], Hart [4] and Korpela [5].
3. The conclusions drawn by Gerushuni and Zhukhovitskii [3] have been modified in light of the more extensive results presented in the present paper.

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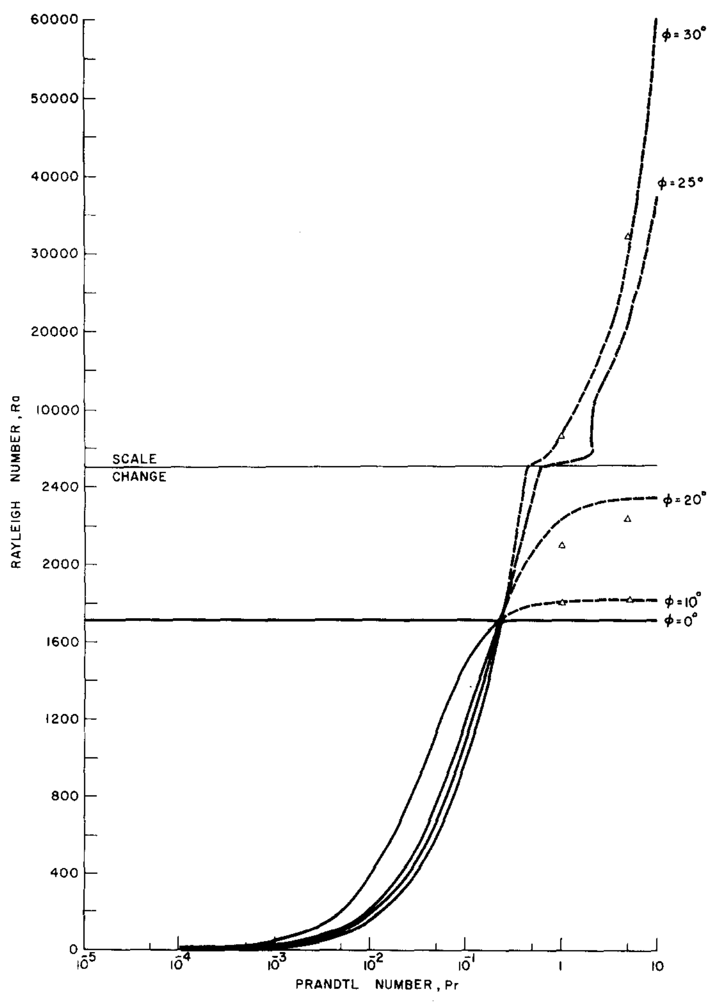

FIG. 1. Gr_c for low angles (—— physically realized, ---- not physically realized, present study; Δ [3], ϕ = 10°, 20° and 30°).

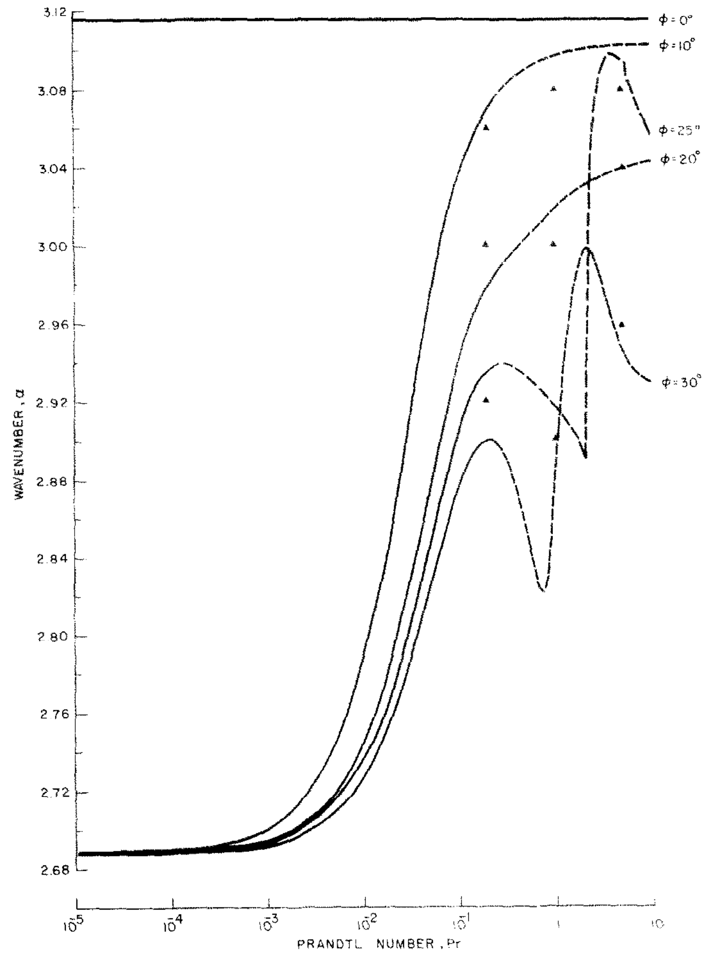

FIG. 2. α_c for low angles (—— physically realized, - - - - not physically realized, present study; ▲ [3], $\phi = 10°$, $20°$ and $30°$).

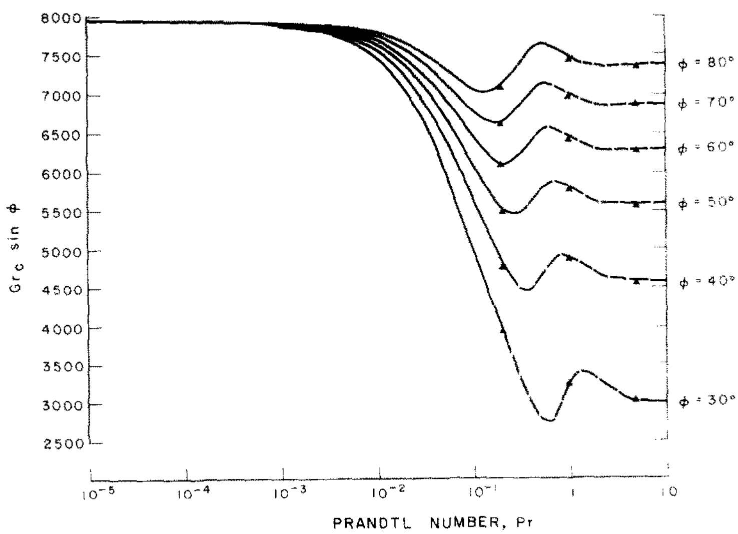

FIG. 3. $Gr_c \sin\phi$ for intermediate and high angles (— — physically realized, - - - not physically realized, present study; ▲ [3]).

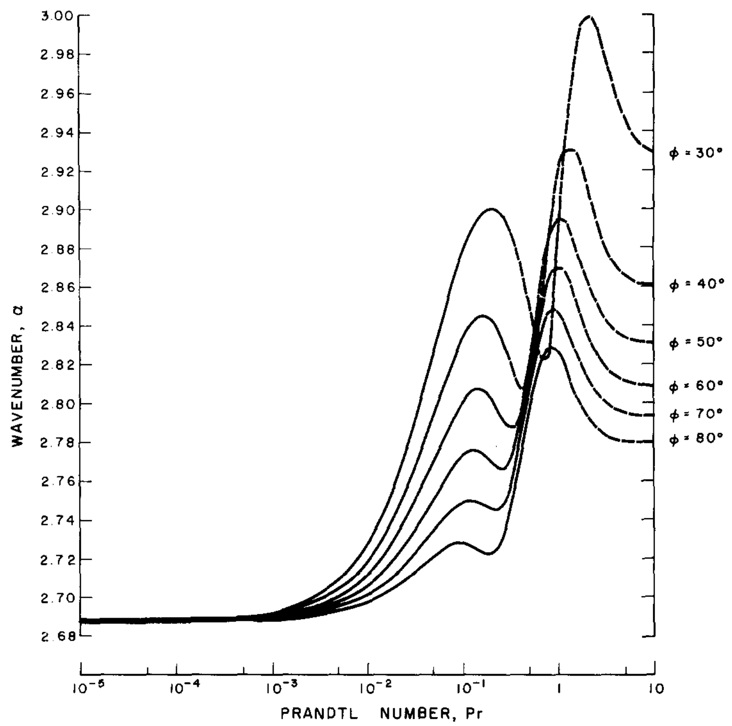

FIG. 4. α_c for intermediate and high angles (—— physically realized, ---- not physically realized).

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